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The Combination of Algebraic and Geometric Methods for Solving Problems in the Course of Geometry

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Annotation: In this paper, the study. 1) the integration of algebraic and geometric methods acts as a means of forming students' ideas about mathematics and its methods;

2) the developed methodology is aimed at developing the creative abilities of students, since having mastered the geometric method, they get the opportunity to choose a method for solving an algebraic problem in accordance with the peculiarities of their thinking style; during the experiment, some students mastered the geometric method better than the algebraic one, which indicates that this method is more accessible to them than the algebraic method.

Keywords: Vector coordinates method, graphical method, method of equations and inequalities.

Introduction.

The trends of the development of modern society, its globalization and total informatization, the rapid growth of information flows and the rapid development of computer technologies, are spending all spheres of the social structure, including one of the main achievements of civilization – education.

At the same time, the global problems facing humanity today (the creation of new energy resources, overcoming the threat of increasing environmental pollution, etc.) require more intellectual forces and highly developed thinking to solve them. In achieving these qualities of personality, as is known, mathematics plays a unique role, which is (and has always been) the foundation of general education.

Moreover, in modern conditions, not only deep mathematical knowledge is necessary, but, first of all, possession of a mathematical method of cognition of the surrounding world. Therefore, the development of the scientific foundations of teaching this method and its main types – algebraic and geometric-is one of the most urgent tasks of the mathematics methodology.

The purpose of the research is to develop an analysis of the current goals and objectives of general secondary and mathematical education, teaching mathematics at the academic lyceum, as well as the analysis of psychological and pedagogical, and educational literature on mathematics to establish the prerequisite for the integration of algebraic and geometric methods in academic mathematical education.

Scientific novelty.

The research consists in the fact that in it the problem of improving general mathematical education is solved on a fundamentally new basis – the concept of integration of algebraic and geometric methods as a process of their combination or connection, carried out by a student by translating educational information from algebraic to geometric languages or from geometric to algebraic and vice versa.

The approbation and implementation of the research results were carried out during purposeful and systematic work with school teachers at scientific and methodological seminars and advanced training courses for educational workers on the basis of the Tashkent Economic University of Education, in the process of teaching mathematics to students of secondary schools (31,33), the academic lyceum at the Tashkent University of Economics; when working with students of the University of Economics within the mathematical circle.

With the study of the vector and coordinate methods, the possibilities of integrating algebraic and geometric methods are expanding. The works of Sh.A.Saipnazarov, G. N. Sarantsev, V. A. Gusev, Yu. M. Kolyagin, G. L. Lukankin and others are devoted to the methodology of teaching the solution of geometric problems by the vector method [33,34 et al.].

Let's consider the blocks of problems that allow integrating the vector method with some geometric methods for solving them.

Example 1. $ABCDA_1B_1C_1D_1$ paralleopiped collars AC and DC_1 diagonals. AC on a straight line M point and DC_1 on a straight line N points $MN \parallel BD_1$ prove the existence and uniqueness of these points if the condition is satisfied. $MN : BD_1$ find the proportions.

Lemma 1. A_1 and A_2 points O let there be different points of a straight line. Top and bottom a and b straight lines to be parallel $\overline{A_1A_2}$ and $\overline{B_1B_2}$ it is necessary and sufficient that the vectors are collinear, that is, such λ for the number

$$\overline{BB_1} = \lambda \overline{A_1A_2}$$

equality is reasonable.

Now we solve the 1-th example.

Let's say, M location *Memory* of the straight line N

location while C_1D (fig - 1) get the points of the straight line.

MN and BD_1 Lemma 1 for parallel straight lines according to,

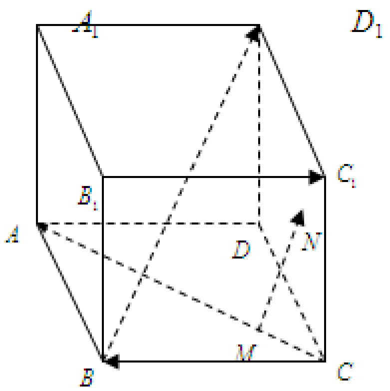

Figure 1. $\overline{MN} = \lambda \overline{BD_1}$ (1)

$\overline{MN}$ and $\overline{BD_1}$ vectors $\overline{CD} = \overline{a},\ \overline{CB} = \overline{b},\ \overline{CA_1} = \overline{C}$

we spread by vectors.

$\overline{BD_1} = \overline{BC} + \overline{CC_1} + \overline{C_1D_1}$, in this $\overline{BC} = -\overline{CB} = -b,$

$\overline{C_1D_1} = \overline{CD} = \overline{a}$, so

$\overline{BD_1} = \overline{a} - \overline{b} + \overline{c}$ (2)

$\overline{MN}$ we write the vector in the following aggregate vector form:

$\overline{MN} = \overline{MC} + \overline{CC_1} + \overline{C_1N}$

Here $\overline{MC}$ vector $\overline{CA}$ collinear to vector, so $\overline{MC} = \lambda \overline{CA}$. However $\overline{CA} = \overline{a} + \overline{b}$ to say , $\overline{MC} = \lambda \overline{a} + \lambda \overline{b}$. $\overline{C_1N}$ vector $\overline{C_1D}$ kollinear on the vector $\overline{C_1D} = \overline{a} - \overline{c}$ to say , , $C_1T = y\overline{C_1D} = y\overline{a} - y\overline{c}$. From this we find the following:

$$\overline{MC} = (x+y)\overline{a} + x\overline{b} + (1-y)\overline{c} \qquad (3)$$

(2) and (3) we put the spreads on (1) and

$$(x+y)\overline{a} + x\overline{b} + (1-y)\overline{c} = \lambda \overline{a} - \lambda \overline{b} + \lambda \overline{c}$$

we form. This vector equality is equally strong in this system

$$\begin{cases} x+y=\lambda \\ x=-\lambda \\ 1-y=\lambda \end{cases}$$

Take off this system, $\lambda = \frac{1}{3},\ x = -\frac{1}{3},\ y = \frac{2}{3}$ we find that it is. All in all, $MN \,//\, BD_1$ the only one to be M and N as long as there are points.

$\overline{MN} = \frac{1}{3} BD_1$. All in all, $MN : BD_1 = 1:3$.

In this matter, a vector expressing the geometrical content of the matter was switched from equality to a system of three scalar equations that are equally strong to it, and the system was solved. There are such cases when it is possible to find a single variable, a system of equations, and the question posed may not have a solution.

Lemma 2. A, B, C and D any three of the points should not lie on one straight line. In order for all these points to lie on one plane without it, it is so α and β. there must be a number, in which it is necessary and sufficient to fulfill this equation

$$\overline{AD} = \alpha\overline{AB} + \beta\overline{AC} \qquad (4)$$

Proof. Necessity. Let's say, A, B, C and D let the dots lie on one plane. According to the condition of the matter AB and AC straight lines do not fall on top. D and the point is that they lie outside. $\overline{AB}$ and $\overline{AC}$ vectors are not collinear, $\overline{AD}$ the vector lies in a plane with them. Therefore, he $\overline{AB}$ and $\overline{AC}$ it spreads by vectors, that is, so α and β for numbers

$$\overline{AD} = \alpha\overline{AB} + \beta\overline{AC}$$

Adequacy. (4) let Equality be reasonable. $\alpha\overline{AB}$ vector $\overline{AB}$ collinear with vector, so it O by putting the point AB we determine that it lies on a straight line. Similar $\beta\overline{AC}$ vector $\overline{AC}$ we determine that it lies on a straight line. (4) Equality $\alpha\overline{AB}$ vector $\overline{AB}$ not one of the vectors is zero, in fact, if for example, $\alpha\overline{AB} = 0$ if, then (4) Equality A, B, C it turns out that the dots lie in one straight line. All in all, $\alpha\overline{AB} \neq 0$, $\beta\overline{AC} \neq 0$, without it (4) from the equation AD cut $\alpha\overline{AB}$ and $\beta\overline{AC}$ it turns out that the parallelogram, built on vectors, consists of a diagonal. Furthermore D location A, B, C it turns out that the dots lie in one plane together. Lemma proved.

Example 2. M and N points $ABCD$ tetraedr edges Middle points (fig-2), *Simple text adventures interpreter* the point is taken so that on the edge, this $AP : AD = 2 : 3$. MNP having a plane BC in what proportions will the edge be?

Solution. BC on the edge Q no matter what the point is M, N, P and Q no three of the points lie on one straight line. Therefore BC on a straight line Q location MNP to lie on the plane, so that α and β there will be numbers (Lemma 2),

$$\overline{PQ} = \alpha\overline{PM} + \beta\overline{PN} \qquad (5)$$

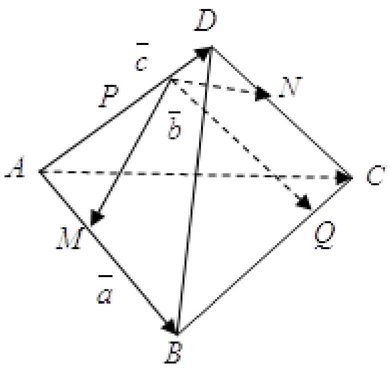

fig-2. We spread these vectors on these vectors.

$\bar{a} = \overline{AB}, \quad \bar{b} = \overline{AC}$, and $\bar{c} = \overline{AD}$

$$\overline{PA} = -\overline{AP} = -\frac{2}{3}\bar{c}, \quad \overline{PD} = \frac{1}{3}\bar{c}, \quad \overline{AM} = \frac{1}{3}\bar{a},$$

$$\overline{DC} = \overline{AC} - \overline{AD} = \bar{b} - \bar{c}, \quad \overline{DN} = \frac{1}{2}\overline{DC} = \frac{1}{2}\bar{b} - \bar{c}$$

$$\overline{BC} = \overline{AC} - \overline{AB} = \bar{b} - \bar{a}$$

Using these,

$$\overline{PM} = \overline{PA} + \overline{AM} = \frac{1}{2}\bar{a} - \frac{2}{3}\bar{c}$$

$$\overline{PN} = \overline{PD} + \overline{DN} = \frac{1}{2}\bar{b} - \frac{1}{2}\bar{c}$$

$$\overline{PQ} = \overline{PA} + \overline{AB} + \overline{BQ}$$

In this sum $\overline{BQ}$ vector $\overline{BC}$ collinear to vector, so it is X there are also such

$$\overline{BQ} = x\overline{BC} = x\bar{b} - x\bar{a},$$

furthermore

$$\overline{PQ} = (1-x)\bar{a} + x\bar{b} - \frac{2}{3}\bar{c}$$

putting the formed expressions (5) into equality

$$(1-x)\bar{a} + x\bar{b} - \frac{2}{3}\bar{c} = \frac{\alpha}{2}\bar{a} + \frac{\beta}{2}\bar{b} - \left(\frac{2}{3}\alpha + \frac{1}{6}\beta\right)\bar{c}$$

we form. This spread is unique

$$\begin{cases} 1 - x = \frac{\alpha}{2}, \\ x = \frac{\beta}{2}, \\ \frac{2}{3} = \frac{2}{3}\alpha + \frac{1}{6}\beta \end{cases}$$

From this system $x = \frac{2}{3}$ we find. All in all, $\overline{BQ} = \frac{2}{3}\overline{BC}$, Q location BC lies on the edge, that is MNP the act of having a flatness BC cut out the edge and cut it V starting from the point when counting $2:1$ it is in proportion.

Coordinate method.

Example 3. The length of the cube edge O equal to. Find the distance between the skew lines diagonals of his two army collars.

Solution. The length of the cube edge a equal to. Find the distance between the skew lines diagonals of his two army collars.

Solution. Reading the coordinates we select as shown in the picture (Figure 3).

Ac And A_1B let the army be asked to find the distance between the diagonals of the bear of the collars. We use the theorem about the general perpendicular of the skew lines straight lines. skew lines is right.

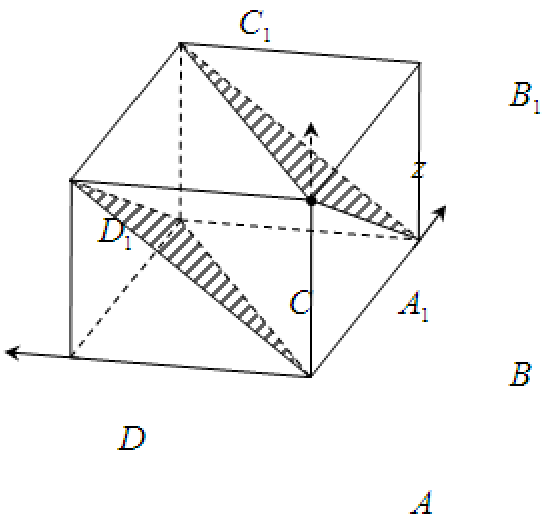

Figure 3.

the general perpendicular of the lines is unique. To find the distance between them, we pass parallel planes, which include these lines. The distance between these parallel planes will be equal to the length of the total perpendicular of the skew lines. Now the mutual parallel is (ACD_1) and (A_1CB) we find the distance between the planes. To do this, we draw up their equations. In relation to the system of coordinates chosen A, C, D_1, A_1, C and B we write down the coordinates of the points.

$$A(o,\ o,\ o),\ \ C(a,\ a,\ o),\ \ D_1(a,\ o,\ a),\ \ A_1(o,\ o,\ a),\ \ C_1(a,\ a,\ a),\ B(o,\ a,\ o)$$

Now (ACD_1) and (A_1C_1B) we compose the equations of the planes. (ACD_1) for the fact that the coordinates have passed through the head, consider its equation $\alpha x + \beta y + \gamma z = 0$ we are looking for in appearance. To this equation C and D_1 putting the coordinates of the points

$$\begin{cases} \alpha a + \beta a = 0 \\ \alpha a + \gamma a = 0 \end{cases} \Leftrightarrow \beta = \gamma = -\alpha$$

we form and put these into the equation above

$\alpha x - \alpha y - 2z = 0$ or $x - y - z = 0$

Now (A_1C_1B) we make plains, we make it

$$\alpha x + \beta y + \gamma z + d = 0$$

we are looking for in appearance:

$$\begin{cases}\gamma a+d=0\\ \alpha a+\beta a+\gamma a+d=0\\ \beta a+d=0\end{cases}\Leftrightarrow\begin{cases}\gamma=-\dfrac{d}{a}\\ \beta=-\dfrac{d}{a}\\ \alpha=\dfrac{d}{a}\end{cases}$$

$\frac{d}{a}x-\frac{d}{a}y-\frac{d}{a}z+d=0$ or

$x-y-z+a=0$

The equation of two planes is found. Now we find the distance between the parallel planes. To do this, we transfer the perpendicular from one optional point of contact to the other, and the distance sought will be the distance between these two parallel planes. For example,

$A(1;\ 1;\ 0)\qquad x-y-z=0$

belongs to the plain. From this point on $x-y-z+a=0$ we lower the perpendicular to the plane, this is the basis of the perpendicular $B(x_0,\ y_0,\ z_0)$ let it be. Thus

$\overline{AB}=(x_0-1,\ y_0-1,\ z_0)$ vector $\overline{n}(1;\ -1;\ -1)$

the vector is collinear.

$\frac{x_0-1}{1}=\frac{y_0-1}{-1}=\frac{z_0}{-1}=\lambda$ or

$x_0=1+\lambda,\ y_0=1-\lambda,\ z_0=-\lambda$

B location $x-y-z+a=0$ for belonging to the plain

$1+\lambda-(-\lambda+1)-(-\lambda)+a=0$

$3\lambda+a=0,\ \ \lambda=-\frac{a}{3}$

$x_0-1=-\frac{a}{3},\ \ y_0-1=\frac{a}{3},\ \ z_0=\frac{a}{3}$

$$|\overline{AB}|=\sqrt{\left(-\frac{a}{3}\right)^2+\left(\frac{a}{3}\right)^2+\left(\frac{a}{3}\right)^2}=\sqrt{\frac{3a^2}{9}}=\frac{a}{\sqrt{3}}$$

Hence, the distance sought $\frac{a}{\sqrt{3}}$ equal to.

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Example 4. The length of the edge is equal to 1 $ABCDA_1B_1C_1D_1$ the cube is given. Cube A from end, DC, BB_1 the middle of the edges and $A_1B_1C_1D_1$ find the radius of the sphere passing through the center of the collar.

Solution. Coordinate head A we select the system of coordinates from, we select the arrows so that the result $B,\ D$ and A_1 points respectively $(1,\ 0,\ 0)$, $(0,\ 1,\ 0)$ and $(0,\ 0,\ 1)$ have coordinates (picture 4). DC and BB_1 coordinates of the middle of the edges accordingly $\left(\frac{1}{2},\ 1,\ 0\right)$, $\left(1,\ 0,\ \frac{1}{2}\right)$

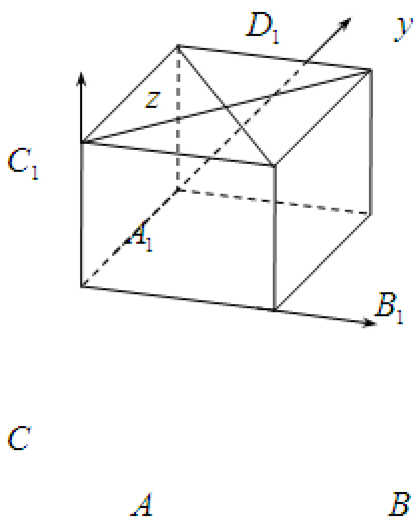


Figure 4

$A_1B_1C_1D_1$ side`s center $\left(\frac{1}{2},\ \frac{1}{2},\ 1\right)$. Centre $(x_0,\ y_0,\ z_0)$ radius R the equation of the sphere in which there is

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$$

This equation can be brought to this view

$$x^2+y^2+z^2+ax+by+cz+d=0$$

For the fact that the sphere is from the beginning of the coordinates $d=0$. $a,\ b$ and C, we form a system of these equations

$$\frac{5}{4}+\frac{1}{2}a+b=0,\ \ \frac{5}{4}+a+\frac{1}{2}c=0,\ \ \frac{3}{2}+\frac{1}{2}a+\frac{1}{2}b+c=0$$

we find it by solving $a=-\frac{13}{14},\ \ b=-\frac{11}{14},\ \ c=-\frac{9}{14}$ this system. So the sphere equation has this appearance:

$$x^2+y^2+z^2-\frac{13}{14}x-\frac{11}{14}y-\frac{9}{14}z=0 \text{ or}$$

$$\left(x-\frac{13}{28}\right)^2+\left(y-\frac{11}{28}\right)^2+\left(z-\frac{9}{28}\right)^2=\frac{371}{28^2}$$

Searched radius $R=\frac{\sqrt{371}}{28}$.

Example 5. Regular rectangular $SABCD$ pyramid SB the side edge formed an angle of 45° with the base plane. At that point SCD find the angle between the collars.

Solution. According to the selected coordinate system B, C, D, S we determine the coordinates of the points. Side of the basis of the pyramid a let it be. Thus

$$B\left(\frac{a}{\sqrt{2}},\ 0,\ 0\right),\quad C\left(0,\ \frac{a}{\sqrt{2}},\ 0\right),$$
$$D\left(-\frac{a}{\sqrt{2}},\ 0,\ 0\right),\ S\left(0,\ 0,\ \frac{a}{\sqrt{2}}\right)$$

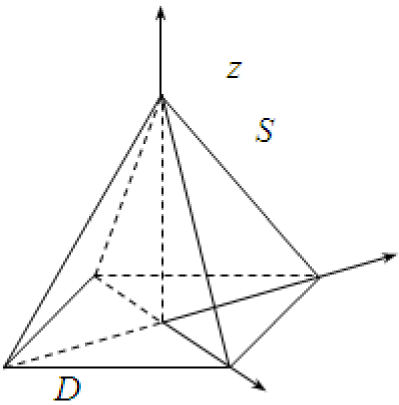


Figure 5

Search angle $\overline{SB}\left(\frac{a}{\sqrt{2}},\ 0,\ -\frac{a}{\sqrt{2}}\right)$ vector and (SCD) is equal to the angle between the plane. (SCD) we make up the plane equation. The equation of the sought-after plane $\alpha x+\beta y+cz+d=0$ let it be visible. $S,\ D,\ C$ the coordinates of the points satisfy this equation

$$\begin{cases}\beta\frac{a}{\sqrt{2}}+d=0\\ -\alpha\frac{a}{\sqrt{2}}+d=0\\ \gamma\frac{a}{\sqrt{2}}+d=0\end{cases}\Leftrightarrow\begin{cases}\alpha=\frac{d}{d}\sqrt{2}\\ \beta=-\frac{d}{d}\sqrt{2}\\ \gamma=-\frac{d}{a}\sqrt{2}\end{cases}$$

Hence, the equation of the sought-after

Plane $\sqrt{2}x-\sqrt{2}y-\sqrt{2}z+a=0 \quad \bar{n}\left(\sqrt{2},\ -\sqrt{2},\ -\sqrt{2}\right)$

Since the angle between the straight line and the plane is equal to the angle between this straight line and its projection in the plane, $\varphi = 90^\circ - \left(\overline{SB} \wedge \bar{n}\right)$ (here it is φ angle between a straight line and a plane).

$$\sin\varphi = \sin\left(90^\circ - \left(\overline{SB} \wedge \bar{n}\right)\right) = \cos\left(\overline{SB} \wedge \bar{n}\right) =$$

All in all,

$$= \frac{\overline{SB}\cdot\bar{n}}{\left|\overline{SB}\right|\cdot\left|\bar{n}\right|} = \frac{\frac{a}{\sqrt{2}}\cdot\sqrt{2}+0\cdot\left(-\sqrt{2}\right)+\left(-\frac{a}{\sqrt{2}}\right)\left(-\sqrt{2}\right)}{\sqrt{\frac{a^2}{2}+\frac{a^2}{2}}\cdot\sqrt{2+2+2}} = \frac{2a}{a\sqrt{6}} = \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$$

$$\varphi = \arcsin\left(\sqrt{\frac{2}{3}}\right)$$

Example 6. The side edges of a triangular regular prism consist of squares. Find the angle between the non-intersecting diagonals of the side edges of the harness (Figure 6).

Solution. AB_1 and BC_1 it is required to find the angle between the diagonals. All edges are equal according to the condition of the matter. In relation to the system of coordinates chosen $A,\ B_1,\ B,\ C_1$ we determine the coordinates of the points, as well as $\overline{AB_1}$ and $\overline{BC_1}$ we find the angle between the vectors, this angle will be the angle we are looking for.

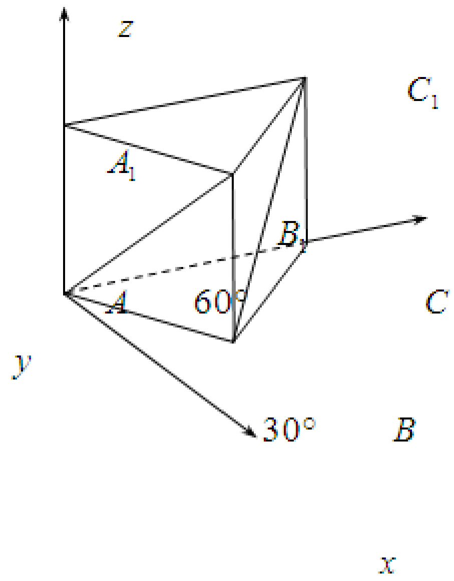


Figure 6

$$A(0,\ 0,\ 0),\quad B\left(\frac{\sqrt{3}}{2},\ \frac{1}{2},\ 0\right),\quad B_1\left(\frac{\sqrt{3}}{2},\ \frac{1}{2},\ 1\right),\quad C_1(0,\ 1,\ 1)$$

(the length of the prism edge was taken as one unit).

$$\overline{AB_1} = \left(\frac{\sqrt{3}}{2},\ \frac{1}{2},\ 1\right),\ \overline{BC_1} = \left(-\frac{\sqrt{3}}{2},\ \frac{1}{2},\ 1\right),\ \left(\overline{AB_1} \wedge \overline{BC_1}\right) = \varphi$$

$$\cos\varphi = \frac{\overline{AB_1} \cdot \overline{BC_1}}{\left|\overline{AB_1}\right| \cdot \left|\overline{BC_1}\right|} = \frac{\frac{\sqrt{3}}{2} \cdot \left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{2} \cdot \frac{1}{2} + 1}{\sqrt{\frac{3}{4} + \frac{1}{4} + 1} \cdot \sqrt{\frac{3}{4} + \frac{1}{4} + 1}} = \frac{1}{4}$$

$$\varphi = \arccos\frac{1}{4}$$

Example 7. In a regular rectangular prism, the ratio of the length of the side edge to the length of the base side is equal to 2. Prizma BD_1 diagonal and (BCD_1) find the angle between the plane (Figure 7).

Solution. We enter the system of coordinates, as shown in Figure 7. The side of the prism base a we mark with, then the side edge length $2a$ will be. $B,\ C_1,\ D$ and D_1 we find the coordinates of the points:

(BCD_1) having a plane $D(0,\ 0,\ 0)$ passes through the point. Hence its equation $\alpha x + \beta y + \gamma z = 0$ it will be visible. To this equation B and C_1 $B(a,\ a,\ 0),\ C_1(0,\ a,\ 2a),\ D(0,\ 0,\ 0)\ D_1(0,\ 0,\ 2a)$

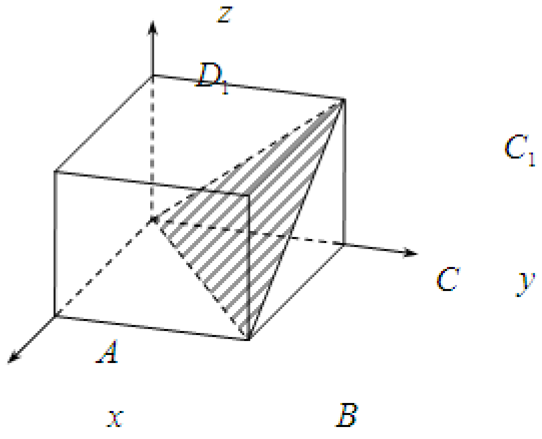


Figure 7

we put the coordinates of the points and get this system.

$$\begin{cases} \alpha a + \beta a = 0 \\ \beta a + 2\gamma a = 0 \end{cases}$$

furthermore $\alpha = 2j,\ \beta = -2j$.

Hence (BC_1D), the equation of the sought-after plane

$2cx-2cy+cz=0$ or

$2x-2y+z=0$

$\overline{BD_1}$ we find vector coordinates $\overline{BD_1}=(-a,\,-a,\,2a)$. (BC_1D) plane normal vector $\overline{n}=(2,\,-2,\,1)$.

BD_1 straight line and BC_1D angle between plane φ we define as, without it

$$\sin\varphi=\frac{\left|\overline{BD_1}\cdot\overline{n}\right|}{\left|\overline{BD_1}\right|\cdot\left|\overline{n}\right|}=\frac{2a}{3\sqrt{6}a}=\frac{2}{3\sqrt{6}}=\frac{\sqrt{6}}{9}$$

$$\varphi=\arcsin\left(\frac{\sqrt{6}}{9}\right)$$

Example 8. Solve a system of equations in natural numbers

$$\begin{cases} x^2+y^2=z^2 \\ \dfrac{xy}{2}=x+y+z \end{cases}$$

Solution. I (we apply the geometry method in combination with the algebraic one)

The first equation of the system can be represented as a right triangle with cathets x, y and the hypotenuse z.

The second equation of the system means that, the area of a right triangle is equal to the perimeter of the triangle. From the first equation systems, we will get that $x=m^2-n^2$, $y=2mn$,

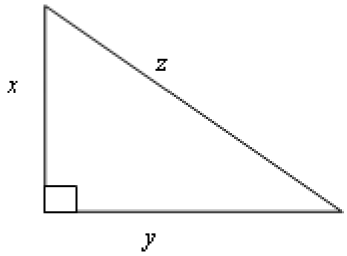


fig. 8

$z=m^2+n^2$, where $m.,n\in N$ and $m>n$. These values are x, y and z we prepare the second equation of the system.

Then $\dfrac{(m^2-n^2)\cdot 2mn}{2}=m^2-n^2+2mn+m^2+n^2$

$(m^2-n^2)mn=2m^2+2mn$;

$(m^2-n^2)n=2m+2n=2(m+n)$;

$(m-n)(m+n)\cdot n = 2(m+n);$

$(m-n)n = 2$

1) $m-n=1,\ n=2$

$m=3,\ n=2$

2) $m-n=2,\ n=1,\ m=3, n=1$

Means, $x = m^2 - n^2 = 3^2 - 2^2 = 5$

$y = 2mn = 2\cdot 3\cdot 2 = 12$

$z = m^2 + n^2 = 3^2 + 2^2 = 13$

1) $(5;12;13),\quad (12;5;13)$

2) $x = m^2 - n^2 = 3^2 - 1^2 = 8$

$y = 2mn = 2\cdot 3\cdot 1 = 6$

$z = m^2 + n^2 = 3^2 + 1^2 = 10$

$(8;6;10),\quad (6;8;10)$

II. The algebraic method

From the second equation of the system, we find $z = \frac{xy}{2} - (x+y)$ and prepare the first equation of the system.

$$x^2 + y^2 = \left[\frac{xy}{2} - (x+y)\right]^2 = \frac{x^2y^2}{4} - xy(x+y) + (x+y)^2 =$$

$$= \frac{x^2y^2}{4} = xy(x+y) + x^2 + 2xy + y^2$$

$$\frac{x^2y^2}{4} - xy(x+y) + 2xy = 0; \qquad xy \neq 0$$

then $\frac{xy}{4} - (x+y) + 2 = 0;$

$xy - 4x - 4y + 8 = 0;$

$xy - 4x - 4y = -8;$

$xy - 4x - 4y + 16 = 8$

$(x-4)(y-4) = 8$

1) $\begin{cases} x-4=8 \\ y-4=1 \end{cases}$ 2) $\begin{cases} x-4=4 \\ y-4=2 \end{cases}$ 3) $\begin{cases} x-4=1 \\ y-4=8 \end{cases}$ 4) $\begin{cases} x-4=2 \\ y-4=4 \end{cases}$

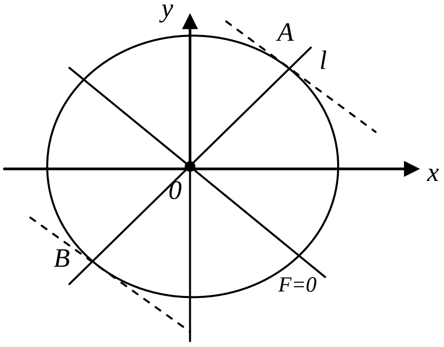
$x=12$	$x=8$	$x=5$	$x=6$
$y=5$	$y=6$	$y=12$	$y=8$
$z=13$	$z=10$	$z=13$	$z=10$

Example 9. Determine the largest and smallest values of the function

$F=3x+4y$ and the condition

$$x^2+y^2=9$$

Solution. I (Geometric method)



The lines of the function level $F=3x+4y$ are parallel lines with an angular coefficient $K=-\frac{3}{4}$, it is obvious that the largest and smallest values are at points A and B. We will find the coordinates it is enough to make the equation of the line l

fig. 9

and solve the system consisting of the equation of the line and the circle.

Note that the line l is perpendicular to the line levels, and, consequently, its angular coefficient $k_1=\frac{4}{3}$ $(k_1\cdot k=-1)$ the straight *line* l passes through the point O and has an angular coefficient $k_1=\frac{4}{3}$ of. Therefore, its equation is as follows; $y=\frac{4}{3}x$. Solving the system

$$\begin{cases} x^2+y^2=9 \\ y=\frac{4}{3}x \end{cases}$$

we get: $x=\pm1{,}8;\ \ y=12{,}4$

$A(1{,}8;\ 2{,}4),\ \ B(-1{,}8;-2{,}4)$

$$F_{\max}=F(1{,}8;\ 2{,}4)=3\cdot1{,}8+4\cdot24=5{,}4+9{,}6=15$$

$$F_{\min}=F(-1{,}8;-2{,}4)=-15$$

II. The algebraic method

$$(3x+4y)^2+(4x-3y)^2=9x^2+24xy+16y^2+16x^2-24xy+9y^2=$$
$$9\left(x^2+y^2\right)+16\left(x^2+y^2\right)=25\left(x^2+y^2\right)=25\cdot9=325$$

$$(3x+4y)^2=225-(4x-3y)^2\le225$$

$$(3x+4y)\le15\Leftrightarrow-15\le3x+4y\le15$$

$F_{\max} = 15,\ F_{\min} = -15$

III. The trigonometric method

Write this equation of the circle in parametric form

$x = 3\cos t,\ y = 3\sin t$. Then

$F = 3x + 4y = 9\cos t + 12\sin t = \sqrt{9^2 + 12^2}\sin(t+\varphi) = \sqrt{225}\sin(t+4) = 15\sin(t+\varphi)$, where

$\varphi = arctg = \frac{3}{4}$

by $\sin(t+\varphi) = 1, \quad F_{\max} = 15$

by $\sin(t+\varphi) = -1, \quad F_{\min} = -15$

IV. The method of Lagrange Multipliers

Suppose $F(x,y)$ and $G(x,y)$ are functions whose first-order partial derivatives exist. To find the relative maximum and relative minimum of $F(x;y)$ subject to the constraint that $G(x,y) = k$ for some constant k, introduce a new variable λ and solve the following three equations simultaneously:

$F'_x(x,y) = \lambda G'_x(x,y) \qquad F'_y(x,y) = \lambda G'_y(x,y) \quad G(x,y) = k$

The desired relative extreme will be found among the resulting points (x,y). Now well go back to our example. Let $G'(x,y) = x^2 + y^2$ and use the partial derivatives

$F'_x = 3, \qquad F'_y = 4, \quad G'_x = 2x$ and $G'_y = 2y$

to get the three Lagrange equations

$3 = 2\lambda x, \quad 4 = \lambda y$ and $2\lambda = \frac{4}{y}$

which implies that $\frac{3}{x} = \frac{4}{y}$ or $y = \frac{4}{3}x$ now substitute $y = \frac{4}{3}x$ into the third equation to get

$x = \pm 1{,}8 \qquad y = \pm 2{,}4$

$F(-1{,}8;\,-2.4) = -15$ and $F(1{,}8;\,2{,}4) = 15$ it follows that when $x^2 + y^2 = 9$, the maximum value of $F(x,y)$ is 15, which occur at the point $(1{,}8;2{,}4)$ and the minimum value is -15, which occurs at $(-1{,}8;-2{,}4)$.

Problem. Find the largest and the smallest values of $F = 3x + 4y$ subject to following inequalities:

$$\begin{cases} x + 3y \le 15 \\ 4x + 3y \le 24 \\ x \ge 0,\ y \ge 0 \end{cases}$$

Lemma: If a linear program has an optimal solution (maximum or minimum), it must occur at u corner point of the feasibility region.

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Geometric method for solving a linear program with two solution variables

1. Graph the feasibility region R and find the coordinates of all corner points of R
2. Make a table evaluating the objective function F at each corner point.
3. If R can be contained in a circle it is bounded. In this case, the largest (smallest) value of F on R is its largest (smallest) value at a corner point.
4. If R cannot be contained in a circle, it is unbounded, and an optimum solution may not exist. However, if it does, it must occur at a corner point.

Now we'll go back to our example.

Solution. First, use the method developed to sketch the feasibility region R (Fig 10.). Note that R is bounded and that the corner points of R is bounded and that the corner points of R are of $(0,0)$, $(0,5)$, $(3,4)$, and $(6,0)$. Evaluating F at the corner points, we obtain the following table:

Corner point	value of $F = 3x + 4y$	
$(0, 0)$	0	← smallest value
$(0, 5)$	20	
$(3, 4)$	25	← Largest value
$(6, 0)$	18	

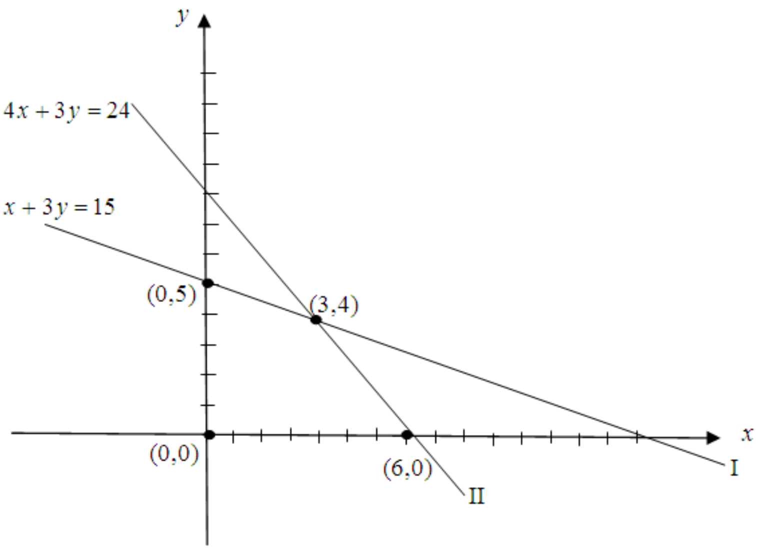

Fig 10.

Thus, the largest value of F is 25 at (3,4), and the smallest value is 0 at the origin.

Problem. Maximize $F = 1{,}2x + y$ subject to the constraints

$$10x + 12y \leq 1920$$
$$5x + 3y \leq 780$$
$$x \geq 0,\ y \geq 0$$

Solution. We find the corner points of the given problem. Evaluating the function $F = 1{,}2x + y$ at the corner points, we obtain the following table:

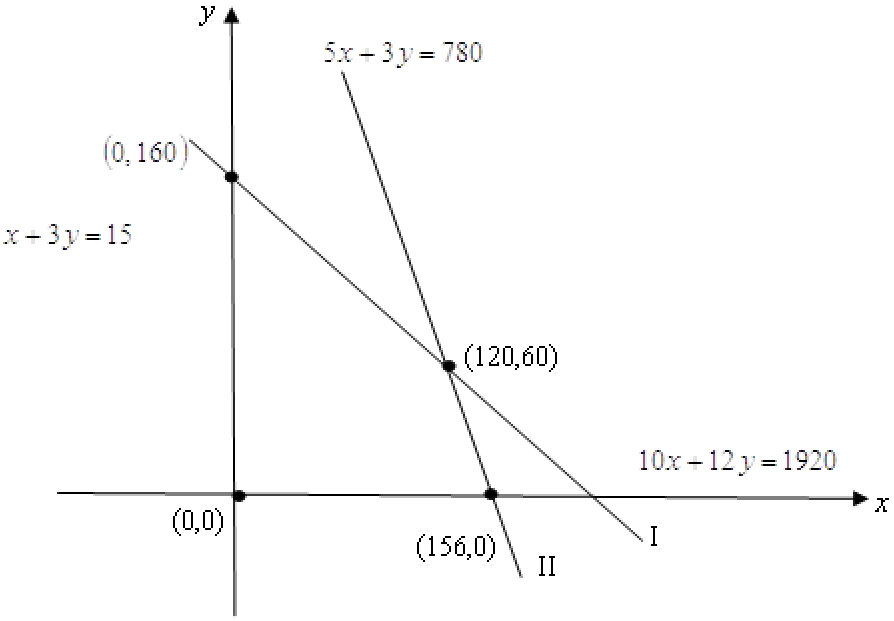

Fig 11.

Corner point	value of $F = 1{,}2x + y$	
$(156,\ 0)$	187,2	
$(120,\ 60)$	204	← Largest value
$(0, 60)$	160	
$(0,\ 0)$	0	

From thus table, we see that the largest value of the objective function over the feasibility region occurs at the point (120, 60)

The experiment was conducted in two institutions: in the secondary school No. 30 of the Sergeli district of the city of Tashkent, 30 students from the 9th and 10th grades were selected (an observation group); in the academic lyceum at TSEU, 30 students from the 2nd and 3rd courses were selected (an experimental group). Without experimental training using the new method in

September 2020, we conducted written control work in these groups. They were offered, mainly, geometric problems, in solving which algebraic tools are used. The control works are made up of the following options. 1, 4, 6

The results of the evaluation of control works are given:

Tables 1

Criteria / Selections	Nedovl.	Udovl.	Well	Great
School №. 30 30	5	14	8	3
academic lyceum 30	3	13	10	4

For four months (II and III quarters), the experimental group (in the academy) conducted training on a new method, outlined the application of algebraic methods to geometric problems, gave illustrative examples about the dependence of geometry and algebra. The students took all the novelties with great favor, easily applied algebraic methods to solving geometric problems and vice versa.

The control work carried out in March 2021 gave the following result:

Tables 1

Categories / Selections	Nedovl.	Udovl.	Well	Great
School №. 30 30	4	14	8	4
academic lyceum 30	1	8	13	8

Stochastic data development is performed according to the criterion χ^2 (chi-square), which is used to compare the distribution of two aggregates according to the state of a certain property based on measurements by the school of names of this property in two independent samples.

From the two aggregates, we will make samples of the volume n_1 and n_2 (in the work $n_1 = 30; n_2 = 30$). The results of measuring the state of the studied properties in the objects of each sample are distributed into m categories (in our case $m = 4$). Thus, we get a dimension table $2 \times m$:

Categories / Selections	1	2	…	m
n_1	a_{11}	a_{12}	…	a_{1m}
n_2	a_{21}	a_{22}	…	a_{2m}

If we denote the probability that an object randomly selected from *the* j –th population $(j = 1, 2)$ will belong *to the* i – th category of the measurement scale $(i = \overline{1, m})$ of the property being checked, by p_{ji}, then we can test the null hypothesis H_0 of equality of probabilities, i.e. $H_0 : p_{1i} = p_{2i}, i = \overline{1, m}$. There is also an alternative hypothesis $H_1 : p_{1i} \neq p_{2i}, i = \overline{1, m}$.

To test the null hypothesis using a criterion χ^2 based on Table 3, the value of statistics is calculated T according to the formula:

$$T = \frac{1}{n_1 n_2} \sum_{i=1}^{m} \frac{(n_1 a_{2i} - n_2 a_{1i})}{a_{1i} + a_{2i}} \qquad (*)$$

Distribution of statistics T is approximated by a distribution χ^2 with $m-1$a degree of freedom $(\vartheta = m-1)$. The significance level χ and value are accepted T is compared with the critical value $\chi_{1-\alpha}$. For a given α value $\chi_{1-\alpha}$, tabulated, [3].

The comparison is carried out at the accepted level of significance $\alpha = 0{,}05$with a "degree of freedom" $\vartheta = m-1 = 4-1 = 3$.

If $T > \chi_{1-\alpha}$, then there are no sufficient grounds for rejecting the null hypothesis.

Let's start calculating the value of statistics T_1according to the data in the table (formula(*))

$$T_2 = \frac{1}{30 \cdot 30}\left(\frac{(30 \cdot 3 - 30 \cdot 5)^2}{5+3} + \frac{(30 \cdot 13 - 30 \cdot 14)^2}{14+13} + \frac{(30 \cdot 10 - 30 \cdot 8)^2}{8+10} + \frac{(30 \cdot 4 - 30 \cdot 3)^2}{3+4}\right) =$$

$$= \frac{299}{378} \approx 0{,}79$$

The results show that there are no sufficient grounds to reject the null hypothesis, since $T_1 = 0{,}79 < \chi_{0,95} = 7{,}815$ (Table D of [3]).

Now we calculate the value T_2 of statistics according to Table 2:

$$T_2 = \frac{1}{30 \cdot 30}\left(\frac{(30 \cdot 1 - 30 \cdot 4)^2}{4+1} + \frac{(30 \cdot 8 - 30 \cdot 14)^2}{14+8} + \frac{(30 \cdot 13 - 30 \cdot 8)^2}{8+13} + \frac{(30 \cdot 8 - 30 \cdot 4)^2}{4+8}\right) =$$

$$\frac{27536}{4620} = 5{,}96$$

And so, after the experiment $T_2 = 5{,}96 < \chi_{0,65} = 7{,}815$, it helps to accept the null hypothesis, since there are no sufficient grounds to reject it.

It should be noted that before the experiment and after it, the null hypothesis took place, i.e. the probabilities of students mastering a new topic are equal to each other. However, if we compare the data in Tables 1 and 2, we will find a big difference when evaluating the values of students. Moreover, in the first case $T_1 = 0{,}79$ and in the second case $T_2 = 6{,}45$, it indicates a large difference between the acquired knowledge of the students of the observed group and the experimental group. In turn, this shows the effectiveness of the conducted experiment.

Variant 1

1. At the base of the triangular pyramid *of the ABSD* is a regular triangle *ABC* with a side equal to one. Edge *The AD* is perpendicular to the plane of the base, and its length is equal to one. Period *M*-middle *VD*. Via a direct line *MS* parallel to the height Triangle AM*ABC* has a plane drawn. Determine the value of the angle between this plane and the plane *AVD*.
2. Solve the system of equations

$$\begin{cases} x+y+z=1, \\ x^2+y^2+z^2=\frac{1}{3} \end{cases}$$

3. The triangle ABC is given. Prove that $\cos A+\cos B+\cos C\le\frac{3}{2}$ where A, B, With the inner angle of the triangle.
4. The triangle ABC is given. Prove that $\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}\le\frac{3}{2}$.
5. Determine the largest value of the function $F=3x+4y$ under the condition $x^2+y^2=4$.

Variant 2

1. Express the radius R of the circumscribed circle in terms of the radius r of the inscribed circle and the angles α, β, j ,
2. The edge of the cube $ABCD\,A_1B_1C_1D_1$ is equal *to a*. Find the radius of the sphere defining through the vertices A and C_1.
3. In a parallelogram, *the ABCD* diagonals are proportional to the sides. i.e. $AB:AD=BD:AB=k$. What values can k *take*? What can be the attitude of the parties?
4. A straight line L and two points are given A and B in out of it. Find a point N *on the line* for which AN^2+BN^2 the smallest value reaches.
5. A rectangle is inscribed in a sector with a central angle of 60^0 and a unit radius (all vertices on the border of the sector). What is the largest value of the area of such rectangles?

Variant 3

1. Determine the largest value of the function $F=2x+y$ under the condition $x^2+y^2=36$.
2. In a right triangle, the catets a and $b$are given . Find the distance from the vertex of the right angle to the nearest point of the inscribed circle.
3. In a trapezoid, the diagonals are 3 4 5, and the segment connecting the midpoints of the bases is 2. Find the area of the trapezoid.
4. The trapezoid is defined by a system of inequalities:

$$\begin{cases} y\le 2x+2 \\ y\ge x-4 \\ x\ge -3 \\ x\le 2 \end{cases}$$

Find the trapezoid parameter.

5. The trapezoid is defined by a system of inequalities:

$$\begin{cases} y \geq 0 \\ y \leq 4 \\ x \geq -x-5 \\ x \leq -3 \end{cases}$$

Find the area of the trapezoid.

Variant 4

1. The trapezoid is defined by a system of inequalities:

$$\begin{cases} y \leq 2x+6 \\ y \geq 2x+4 \\ x \leq 0 \\ y \geq 0 \end{cases}$$

Find the perimeter of the trapezoid.

2. Find the area of the common part of two squares, if each side is equal *to a* and one is obtained from the other by turning around the vertex at an angle of 45^0.

3. Find the corners of the rhombus if the area of the circle inscribed in it is half the area of the rhombus.

4. Determine the acute angle of the rhombus, in which the side is the geometric mean of its diagonals.

5. The height of a regular triangular pyramid is equal to N. The dihedral angle between the side faces is equal γ to . Find the volume of the pyramid.

Conclusion.

In this work, the study of the possibilities of integrating algebraic and geometric methods in general mathematical education and the development of an appropriate teaching methodology were carried out in the mainstream of systems analysis with the involvement of data from various scientific fields general and educational psychology, didactics, history of mathematics, etc.

In the course of our research, the hypothesis was substantiated and confirmed: if the algebraic and geometric methods are considered in teaching as methods of cognitive activity of students, based respectively on the system of algebraic and geometric knowledge, then this will make it possible to classify these methods, then, using a systematic approach, isolate the components and geometric methods, explaining the mechanism of this process in general mathematical education, in addition, if you build a methodological system that includes the student's personality, goals, content, methods, forms and means of integrating algebraic and geometric methods, develop the conditions for its functioning in the school educational process and introduce them into practice, this will improve the quality of knowledge and ideas about mathematics and its methods, to develop the creative mathematical abilities of students through the use of the algebraic method in geometry, and the geometric method in algebra, take into account the individual characteristics of students associated with types of thinking: logical – verbal and spatial – like.

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