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Extremes in Problems of Physics

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Annotation: This article discusses methods for solving problems in physics. It is clear that among the various physical problems there are those in which the extreme values of the required quantities are determined (the minimum coefficient of friction, the maximum angle of inclination, etc.). Therefore, ways of walking extrema are shown, and it is also said that there is a fairly universal method based on the use of differential calculus, but it is not always the simplest. Several examples have been considered.

Keywords: Extremum, differential, trigonometry method, analytical method, maximum and minimum conditions.

1. Introduction

Among the various physical problems, there are those in which the extreme values of the required quantities are determined (the minimum coefficient of friction of friction, the maximum angle of inclination, etc.). Often in such cases, the result is simultaneously influenced by several competing factors, some of which contribute to its increase, while others decrease it. It in this case, due to some changes, the decisive influence passes from one factor to another, then the desired value first increases and then decreases, or vice versa. In the first case, it has a maximum, in the second – a minimum. Methods for finding extremums depending on specific conditions in problems can be different. There is a fairly universal method based on the use of differential calculus, but it is not always the simplest. In some cases, trigonometric or geometric methods, as well as graphs, can be useful. In a word, options are possible. Let's look at a few examples.

Task 1. What is the minimum force required to pull the rope in order to uniformly move a sled of mass *m* kg along horizontal asphalt if the coefficient of sliding friction is *k*? $m = 10\,kg,\ k = 0{,}70$

Solution. Let's write down the equations of motion of the sled in projections on the horizontal and vertical directions (Fid.1):

$$-F_{fr} + F\cos\alpha = 0,$$
$$-mg + N + F\sin\alpha = 0$$

Where α - is the angle between the rope and the horizon, and the friction force is $F = kN$. From here we find the tension of the rope:

$$F = \frac{kmg}{\cos\alpha + k\sin\alpha}$$

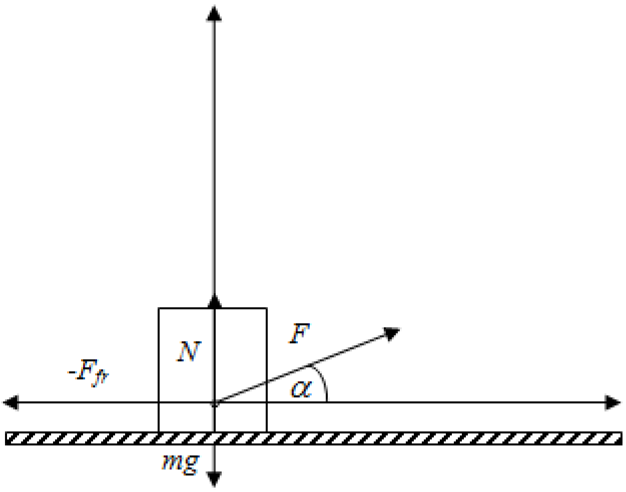

Fig.1.

Let us analyze the dependence of the force F on the angle F on the angle α . The sled will move uniformly if the horizontal component of the rope tension force $F\cos\alpha$ is equal to friction force F_{fr} . Therefore, to ensure the minimum force F , the rope, it would seem, must be pulled horizontally, i.e. at an angle $\alpha = 0$. But on the other hand, it is desirable that the angle α be larger, since in this case, due to the increase in the vertical component $F\sin\alpha$, which tends to raise the sled, their pressure on the support decreases and, accordingly, the friction force F_{fr} decreases. This, the result, as we see are influenced by two competing factors. Imagine the dependence $F = F(\alpha)$ in the form of a graph. (Fig. 2).

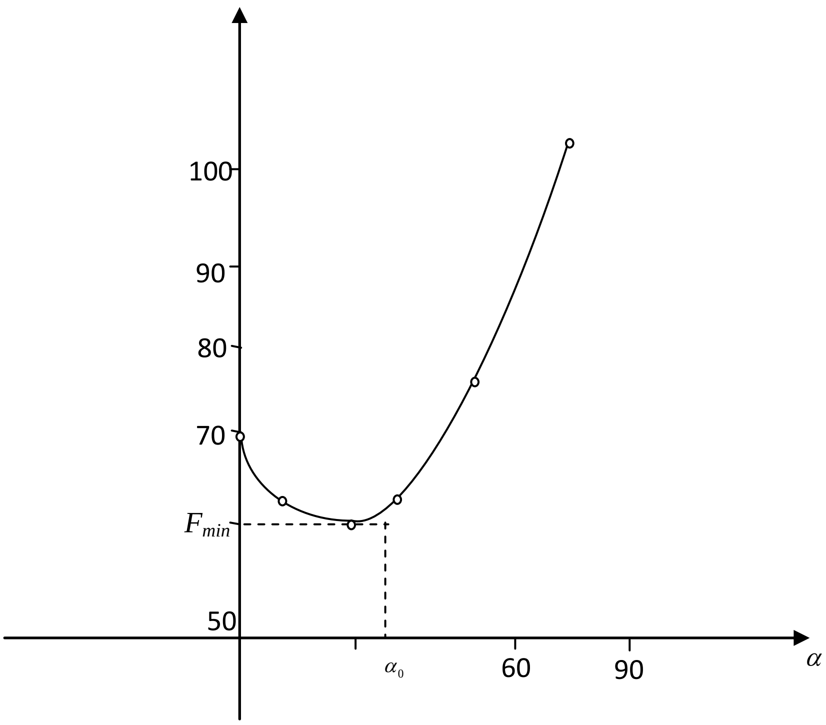

Fig. 2

It shows that the function under study has a minimum.

1-way. The function *F* is minimal if the denominator is maximal. Let's denote it by the lette *Y*, then we get the following function

$$y = \cos\alpha + k\sin\alpha$$

By the trigonometry formula we have

$$y = \cos\alpha + k\sin\alpha = \sqrt{1+k^2}\cos\left(\alpha - arctgk\right)$$

if $\cos\left(\alpha - arctgk\right) = 1$, our function has *a* maximum.

$$\alpha - arctgk = 0, \quad \alpha_0 = arctgk, \quad tg\alpha_0 = k \qquad \alpha_0 = arctg\,0{,}7 \approx 35^0$$

Then

$$F_{\min} = \frac{kmg}{\sqrt{1+k^2}} \approx 56\ N$$

2-way. Find the derivative of *y* from α equate it to zero:

$$y' = -\sin\alpha + k\cos\alpha = 0$$

Hence, denoting the corresponding angle α_0 , we obtain

$$tg\alpha_0 = k, \quad \alpha_0 = arctgk = 35^0$$

Then $F_{\min} = \dfrac{kmg}{\cos\alpha_0 + k\sin\alpha_0}$, which

Using the relations

$$\cos\alpha_0 = \frac{1}{\sqrt{tg^2\alpha_0 + 1}} = \frac{1}{\sqrt{1+k^2}}$$

$$\sin\alpha_0 = \frac{1}{\sqrt{1+tg^2\alpha_0}} = \frac{1}{\sqrt{1+k^2}}$$

we find the desired

$$F_{\min} = \frac{kmg}{\sqrt{1+k^2}} \approx 56\ N$$

Task 2. On a horizontal plane is a large motionless vessel completely filled with water. A stream of water flows out through a small hole in its side wall. At what height should the hole be in order to maximize the range of the jet? What is this range? Vessel height *H*. friction is ignored.

Solution. The flight range of the jet is $s = V_0 t$, and the height of its fall is $h = \dfrac{gt^2}{2}$, where $V_0 = \sqrt{2g\left(H-h\right)}$ the velocity of water outflow from the hole, *t* – is the time of water fall (Fig.3). Hence, eliminating *t*, we obtain

$$s = V_0\sqrt{2h/g} = 2\sqrt{h\left(H-h\right)}$$

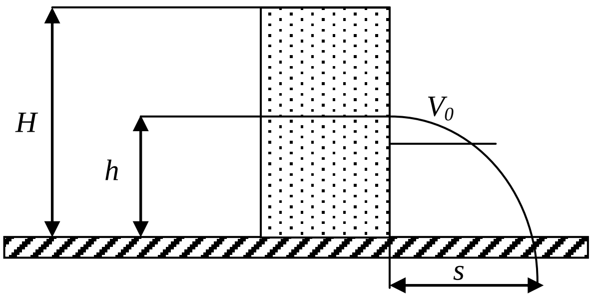

Fig. 3

As expected, the flight range *s* depends on the hole height *h*. Let's analyze this dependence. On the one hand, the lower the hole, the higher the water column above it and, consequently, the greater the water outflow velocity *V*; hence, the flight range *s* must also be greater. But on the other hand, the smaller *h*, the shorter the flight time *t*, which leads to a decrease in range.

The are two factors at play here. Consider the graph of the function

$S = S(h) = 2\sqrt{h(H-h)}$ (Fig. 4). It can be seen from it that *S* is maximum at $h = h_0 = \frac{H}{2}$. In this case $S_{\max} = H$

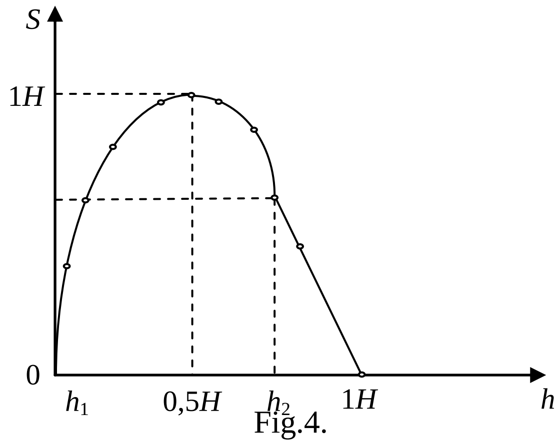

Fig.4.

The same results can be obtained analytically. Indeed, the function $S(h)$ is maximal when the radical expression is maximal. Denote it by the letter *y*, take the derivative of y' with respect to the argument *h* and equate it to zero:

$$y' = H - 2h = 0$$

Hence $h = h_0 = \frac{H}{2}$, and the maximum range.

$$S_{\max} = H.$$

To find the extremum also use the following method. Let's ask ourselves the question – how many heights *h* correspond to a given range *S*? As can be seen from Figure 4, there are two such values for $S < S_{\max}$, one for $S = S_{\max}$, and none for $S > S_{\max}$. To use this observation, we will assume that *S* is given in the equation

$S = 2\sqrt{h(H-h)}$, and h is not known.

Rewriting it as a quadratic equation

$$h^2 - Hh + \frac{S^2}{4} = 0$$

we write down the condition of equality to zero of the discriminant:

$$H^2 - S_{\max}^2 = 0 \ ,$$

whence we obtain the same answer, of course, as before:

$$S_{\max} = H$$

Although in this problem this technique looks somewhat artificial, in other cases it greatly simplifies the solution.

Task 3. The gas was transferred from a state with a pressure $P_1 = 150\ kPa$ and a volume of $V_1 = 20{,}5l$ to a state with a pressure of $P_2 = 400\ kPa$ and a volume of $V_2 = 4{,}5l$.

Determine the maximum temperature of the gas in this process, if it is known that its graph in coordinate P,V is a strait line. The amount of gas is $\vartheta = 0{,}75$moll.

Solution. Let's build a graph of the change in the state of the gas in the coordinates P,V (Fig. 5 a). To determine the maximum gas temperature $T_{\max}$ in this process, we write the equation of the straight 1 – 2:

$P = a - bV$, where $a = P^0, b = tgx$. Let's move on to the variables T,V, using ideal gas equation of state

$$PV = \upsilon RT;$$

$$aV - bV^2 = \upsilon RT$$

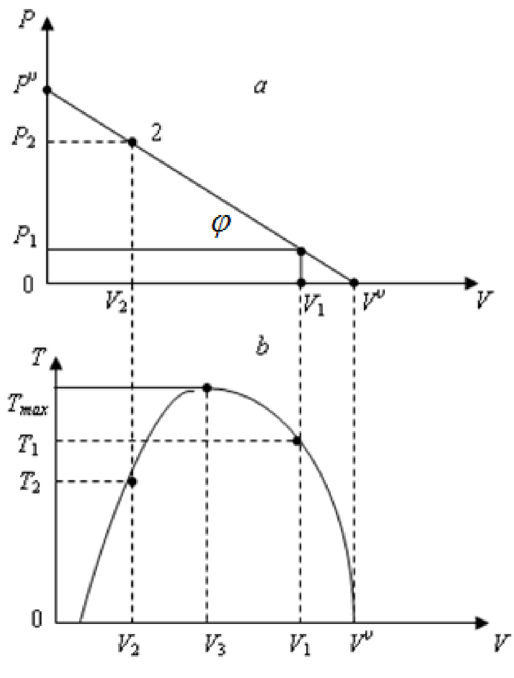

Fig. 5.

This is the equation of a parabola passing through the origin (Fig.5, *b*). Its roots are equal to $V = 0$ and $V^0 = \frac{a}{b}$ and the axis of symmetry passes through the $V^0 = \frac{a}{b}$ and int $V_3 = \frac{a}{2b}$. Volume V_3 corresponds to the required temperature:

$$T_{max} = \frac{a}{\upsilon R}V_3 - \frac{b}{\upsilon R}V_3^2 - \frac{a^2}{4\upsilon Rb}$$

Note that the same equality can be easily obtained by examining the parabola equation for an extremum using derivatives. Now let's find the parameters a and *b* (see Fig. 5 *a*):

$$a = P^0 = P_2 + \left(P^0 - P_2\right) = P_2 + \left(P_2 - P_1\right)V_2 / V_1 - V_2,$$

$$b = tg\varphi = \frac{P_2 - P_1}{V_1 - V_2}.$$

Then finally we get

$$T_{max} = \frac{\left(P_2V_1 - P_1V_2\right)^2}{4\vartheta R\left(P_2 - P_1\right)\left(V_1 - V_2\right)} = 568K$$

Task 4. To study the resonance, a circuit is assembled, shown in the figure. At what oscillator frequency does the voltmeter give maximum readings? What is this maximum voltage equal to if the generator voltage amplitude is $u_0 = 1v$? Now will the maximum readings of the voltmeter change when the resistance *r* kohm decreases to 100 ohm? All elements of the circuit can be considered ideal.

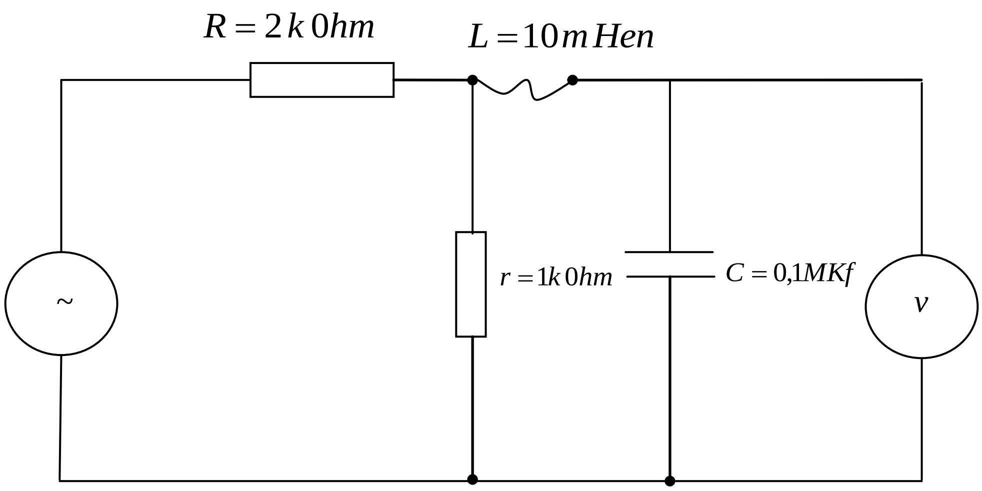

Fig.6.

To significantly simplify the calculations, it is convenient to replace the alternating voltage generator and the two resistors connected to it with the so-called equivalent source with the output voltage amplitude $u_m = \frac{u_0 r}{R + r}$ and internal resistance $Z = \frac{R \cdot r}{R + r}$. Now the circuit turned out to be quite simple, and the amplitude of the voltage across the capacitor can be easily written:

$$u_c = \frac{u_m Z_c}{Z_{tot}} = \frac{u_m}{\omega c\sqrt{Z^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}$$

Let us investigate the obtained frequency function ω to the maximum. To do this, consider the denominator, or better, the square of the denominator, and find out at what value of ω it is minimal.

$$\omega^2 c^2\left(Z^2 + \omega^2 L^2 - 2\frac{L}{c} + \frac{1}{\omega^2 c^2}\right) = \omega^2 c^2 Z^2 + \omega^4 L^2 c^2 - 2\omega^2 Lc + 1$$

We equate its derivative with respect to frequency to zero:

$$\left(\omega^2 c^2 Z^2 + \omega^4 L^2 c^2 - 2\omega^2 Lc + 1\right)' = 0$$

$$2\omega c^2 Z^2 + 4\omega^3 L^2 c^2 - 4\omega Lc = 0$$

Where do we find

$$\omega^2 = \frac{1}{Lc} - \frac{Z^2}{2L^2} = \omega_0^2\left(1 - \frac{Z^2 c}{2L}\right) \quad \left(\omega_0 = \frac{1}{\sqrt{Lc}}\right)$$

It can be seen that at $R = 2k0hm$ and $r = 1k0hm$ $Z = 667k0hm$ and under the root is a negative value. This means that the function does not have maxima and minima at $\omega > 0$ and the maximum voltage of the voltmeter is obtained either at a very high or at a very low frequency. In our first case, it is clear that the maximum will be at zero frequency and the voltmeter will show 0,33V. in the second case, the readings will be maximum at the frequency $\omega = 0{,}95\,\omega_0$, and will be approximately 0,15V.

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