



Probabilistic approach to the calculation of ground and foundations by limit states

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Annotation: The article analyzes the formation and development of the method of calculating building structures and foundations by limiting states, since the development of the method of calculating their reliability (reliability level), in turn, is based on the method of calculating by limiting states. During its development, at the beginning, the coefficients of uniformity, overload, and then reliability coefficients were introduced into the calculation. For a long time, a fairly large amount of statistical information has accumulated on external influences and internal properties of building structures and foundation soils. This made it possible to develop a so-called parametric theory of reliability, in which, in most cases, the distributions of both external influences and internal properties of materials are assumed to obey the normal law. The normal distribution law is confirmed by numerous experimental results. In the case when external and internal factors obey more complex laws, distributions, then in this case it is possible to determine the level of reliability by linearization.

Key words: Building structures, grounds, reliability coefficient, reliability.

Introduction

For a long period, the design of building structures and their foundations was based on calculation methods either for permissible or destructive loads. The basis for ensuring the reliability and efficiency of structures was the concept of the total reserve coefficient, respectively, as the ratio of the permissible or destructive load to the calculated one. Moreover, theoretical and experimental studies did not reveal the structure of the total reserve coefficients, their values were established intuitively – based on the analysis of the ratios of permissible or limit loads and loads of favorably operated structures taken into account. Therefore, the values of the total stock coefficients used in the calculations were empirical in nature and were interpreted by many as "ignorance coefficients". It would seem that since the stock coefficients were assigned large units, therefore, reliable operation of structures and bases should be ensured. However, in reality, there are many examples of damage and accidents of both building structures and buildings and structures designed with this approach. On the other hand, calculations on permissible loads did not always ensure the cost-effectiveness of the decisions taken.



Attempts to provide the required load-bearing capacity of structures and bases by increasing the calculated margin coefficient did not give a full guarantee of reliability. This led, in some cases, to contradictions and significant deviations of the calculated data from the results of actual observations.

Therefore, the searches of many researchers were aimed at revealing the concept of the total margin coefficient as the main criterion for ensuring the reliability and efficiency of structures.

For the first time in 1899 N. I. Psarev formulated and solved a probabilistic and statistical problem: about the error of estimating the average value of the indicator under study and about the number of definitions necessary so that this error does not exceed the permissible limits [1,2].

The statistical nature of the stock coefficients was investigated in 1926 by M. Mayer [3] and in 1929 by N.F. Khotsialov [4]. The latter drew attention to the inevitable dispersion of the cubic strength of concrete laid in the dams of power plants. Considering the cubic strength of concrete as obeying the normal distribution law and assuming the hydrostatic load on the dam to be deterministic, he obtained a formula for the necessary margin of safety, which guaranteed indestructibility with a predetermined security. In this work, some concepts of reliability theory have already been laid down.

An outstanding role in the introduction of statistical methods into construction practice was played by the works of N.S. Streletsky [5,6], the beginning of publication, which dates back to 1935. As random variables, he used not only the strength characteristics of the material, but also the load parameters, in fact, he formulated a static concept of structural reliability, which was implicitly reflected in the method of calculating structures by limit states.

During this period, methods of probability theory also began to be introduced for calculating individual parameters of the variability of natural soils: for example, the works of G.I. Pokrovsky and S.I. Sinelshchikov, published in 1937-38 [7].

Formation of the method of calculation by limit states. In connection with the development of new norms in 1945, it was decided to organize a special commission consisting of V.M. Keldysh, I.I. Goldenblat, N.S. Streletsky, etc. on unification of methods of calculation of building structures [8]. As a result of the work of this commission, the principled foundations of the calculation method for limit conditions introduced in the "Building Regulations" (SNIIP) have been created. In particular, a conditional scheme of calculation coefficients was adopted, according to which the total reserve coefficient is divided into three groups of differentiated coefficients: overload coefficients, taking into account the random variability of loads; uniformity coefficients, taking into account the deviation of the strength characteristics of materials and coefficients of working conditions, taking into account the conditionality of calculation schemes and other factors not taken into account in the calculations directly. The ultimate condition of the structure (or base) was called such that their long-term operation becomes impossible or irrational.

The limit state calculation method played a progressive role for the development of probabilistic methods, since it allowed us to evaluate separately the random nature of the properties of materials and loads. Currently, the limit state method is used in the calculation of all building structures and foundations. A similar method is used abroad, which has been called "semi-probabilistic" [9, 10].

However, according to the limit state method, the coefficients of uniformity and overload are determined separately for each calculated factor, regardless of the variability of other factors.



This leads, in some cases, to an overestimation of the reliability of the structure, in others – to an underestimation. For this reason, recently more and more attention has been paid to the use of differentiated coefficients established on the basis of an objective integral criterion, as which the quantitative characteristic of reliability is recognized. Reliability is a measure of the preservation of the necessary properties of a structure or object and the ability to withstand random factors of various kinds that violate these properties [8]. The concept of reliability is associated with the random nature of the quantities characterizing the operability of the object and is quantitatively revealed by the apparatus of probability theory.

Reliability based on the normal law of distribution of external and internal factors. Reliability research begins with a joint analysis of the distributions of the internal properties of the object and external conditions, as well as the requirements imposed on it. At the same time, all calculated values are divided into two main groups: generalized bearing capacity (internal factor)- Y_1 and generalized load (external factor) - Y_2 . This allows us to formulate the problem of determining reliability in the form of a requirement to fulfill with some probability of inequality:

$$Y_1 - Y_2 \geq 0, \quad (1)$$

which can express the condition of reliability of structures or foundations for any limiting condition, i.e. characterize their bearing capacity, deformation or crack resistance [8,11,12].

Both the external load and the bearing capacity are variable, random variables, so the absolute fulfillment of inequality (I) is meaningless. One can only demand that this inequality be observed with a certain probability close enough to unity.

There are currently two ways to study the reliability of building structures and foundations. The first is the joint study of distributions Y_1 и Y_2 , the second is to study their distribution separately.

The first most general way is implemented if the statistical characteristics of variability are known Y_1 и Y_2 . However, often, due to the lack of necessary information, this path is used only for the purpose of constructing a general theory of reliability [11, 12, 14, etc.]. The second way, being an integral part of the first, due to its importance, has an independent meaning and is used in applications of reliability theory [15, 16, 17, etc.].

When assessing reliability, the behavior of an object over time (its durability) is important. However, at present, the issues of reliability of building structures and foundations are considered, as a rule, without taking into account the time factor (initial reliability) using the so-called parametric reliability theory [8,12,13, etc.]. The issues of redundancy, maintainability, durability for practical use are not sufficiently developed. One of the reasons for this is the lack of systematic statistical information about the necessary parameters.

Consider the joint study of distributions Y_1 и Y_2 according to the parametric theory of reliability.

For the first time, a probabilistic criterion for assessing the reliability of structures based on a joint study of distributions was proposed by N.S. Streletsky [5]. To indicate the probability of trouble-free operation, he introduced the concept of "guarantees of indestructibility".

Taking load distribution probability curves $P(Y_2)$ and bearing capacity $P(Y_1)$, obeying normal laws, N.S. Streletsky breaks off these curves at some point $Y_{1,0} = Y_{2,0}$ (fig. 1a).



Introducing designations for small areas ω_1 and ω_2 for the "guarantee of indestructibility", he got the expression:

$$\Gamma = 1 - \omega_1 * \omega_2, \tag{2}$$

where

$$\omega_1 = \int_0^{y_{1,0}} P(Y_1)dY_1;$$

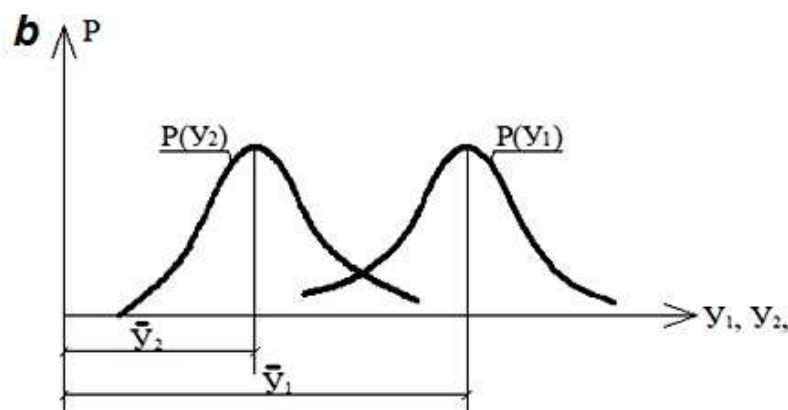
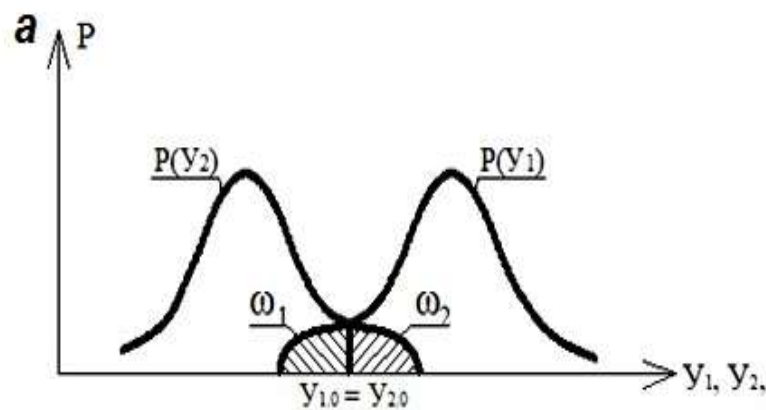
$$\omega_2 = \int_{y_{2,0}}^{\infty} P(Y_2)dY_2.$$

Subsequent comparative calculations given in [11] showed that this criterion does not allow determining unambiguously the probability of indestructibility. Therefore, it is not currently used to determine reliability.

The fundamental works of A.R. Rzhnitsyn contributed to the further development of probabilistic methods for calculating the reliability of building structures [8]. The principal provisions of the results of his research are that the initial calculated data are presented in the form of random variables with given distribution curves. Based on the established deterministic dependencies between generalized internal factors Y_1 and generalized external factors - Y_2 (fig. 1b), their difference is determined

$$Y = Y_1 - Y_2 \geq 0, \tag{1}$$

for which the distribution curve is constructed.



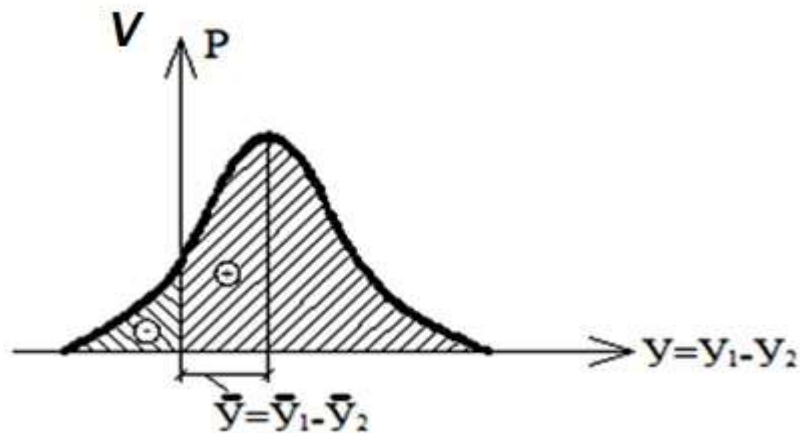


Fig. 1 Calculation schemes for determining reliability.
 a) by N.S. Streletsky, b) and c) by A.R. Rzhantsyn

Note that if Y_1 and Y_2 have normal distributions, then the distribution of their difference Y , it will also be normal (fig. 1v). The probability that the difference will have a positive value is the amount of security or reliability, which should be close enough to one.

Mathematical expectation \bar{Y} and variance σ_y^2 for distribution Y expressed through the appropriate parameters:

$$\bar{Y} = \bar{Y}_1 - \bar{Y}_2; \tag{4}$$

$$\sigma_y^2 = \sigma_{y1}^2 + \sigma_{y2}^2; \tag{5}$$

Where \bar{Y}_1, \bar{Y}_2 - mathematical expectations of the corresponding distributions;

$\sigma_{y1}^2, \sigma_{y2}^2$ - variances of distributions.

The inverse of the coefficient of variability of the distribution Y , A.P. Rzhantsyn called it a "safety characteristic" Z , i.e.

$$Z = \frac{\bar{Y}}{\sigma_y}, \tag{6}$$

where σ_y - the mean square deviation of the generalized factor Y .

Safety characteristics Z implicitly represent reliability. Mathematically, it has the value of a determinant in the formula for calculating the probability of values falling into the negative region (see fig. 1b). For a normal distribution, this probability, i.e. the probability of destruction or failure Q , calculated by the formula

$$Q = \frac{1}{2} - \Phi(Z), \tag{7}$$

where $\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_0^z \exp\left(-\frac{z^2}{2}\right) dz$, It is a Gaussian probability integral, the values of which are tabulated [18, 19].



Since "solvability" and "unsolvability" are opposite events and form a complete group of events, the probability of indestructibility, taking into account expression (7), is determined by the formula

$$H = 1 - \left[\frac{1}{2} - \Phi(Z) \right] = 0,5 + \Phi(Z). \tag{8}$$

Equation (8) with known statistical parameters of the generalized factor – Y allows us to calculate the probability of "indestructibility" H, called by N.N.Ermolaev "reliability" or the level of "reliability" [12].

The total stock ratio according to A.R. Rzhantsyn γ_0 there will be a deterministic value equal to the ratio of the mathematical expectations of internal and external factors, i.e.

$$\gamma_0 = \frac{\bar{Y}_1}{\bar{Y}_2}. \tag{9}$$

Using the concept of variability (coefficients of variation) for distributions Y_1 and Y_2

$$V_{y1} = \frac{\sigma_{y1}}{\bar{y}_1},$$

$$V_{y2} = \frac{\sigma_{y2}}{\bar{y}_2},$$

where σ_{y1} and σ_{y2} the mean square deviations of the corresponding values, and substituting these expressions into equation (6), A.R.Rzhantsyn obtained a safety characteristic in the form

$$Z = \frac{\gamma_0 - 1}{\sqrt{V_{y1}^2 * \gamma_0^2 + V_{y2}^2}}. \tag{10}$$

Expressions (8), (9) and (10) allow us to determine the level of reliability H using the value of the total margin coefficient γ_0 and variations of random variables Y_1 and Y_2 with their normal distribution law.

Reliability under an arbitrary law of distribution of external and internal factors.

Solutions for reliability estimation are also obtained with asymmetric and lognormal distribution law Y_1 and Y_2 . In the case when the generalized load and bearing capacity are distributed according to Pearson's law of the third type, the solutions are obtained by R.A.Muller [20]. More general solutions allowing the distribution of Weibull, Rayleigh and Pearson to be considered as special cases were obtained by P.A.Vizir [21].

To assess the reliability of structures and foundations, as follows from the above, it is necessary to establish the laws of distribution of internal and external factors, as well as the parameters of their variability.

Let's consider a general solution to the problem of determining the parameters of the distribution law by the linearization method.

Suppose that the load-bearing capacity of structures or bases F can be expressed depending on the factors determining it as follows:

$$F = f(x_1, x_2, x_3, \dots, x_n). \tag{11}$$

Next, imagine that the factors x_1, x_2, \dots, x_n are random variables obeying the law of normal distribution. In this case, the function F it will also obey the law of normal distribution.



It is necessary to determine the mathematical expectation \bar{F} variance σ_F^2 functions F , that is, the characteristics of the distribution of this function.

Consider the function F in the neighborhood of her in mathematical expectation. After mathematical transformations, the final results for the mathematical expectation function F have the form [19].

$$\bar{F} = f(\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots, \bar{x}_n); \tag{12}$$

and the variance of the function

$$\sigma_F^2 = \sum_{j=1}^n \left(\frac{\partial F}{\partial x_j} \right)^2 \sigma_{x_j}^2, \tag{13}$$

where $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n$ - mathematical expectations of arguments.

Formula (13) becomes more complicated if the factors x_1, x_2, \dots, x_n , they will depend on each other. In this case, the expression for the mathematical expectation of the function will remain the same, and an additional term will appear in the expression for determining the variance, i.e.

$$\sigma_F^2 = \sum_{i=1}^n \left(\frac{\partial F}{\partial x_i} \right)^2 \sigma_{x_i}^2 + 2 \sum_{i < j}^k \left(\frac{\partial F}{\partial x_i} \right) \left(\frac{\partial F}{\partial x_j} \right) r_{x_i x_j}, \tag{14}$$

Where $r_{x_i x_j}$ - correlation coefficients between arguments;

$\frac{\partial F}{\partial x_i}; \frac{\partial F}{\partial x_j}$ – partial derivatives of the function for each argument:

n- total number of arguments (factors);

r- the number of pairs of correlated arguments.

In the event that the arguments obey distributions other than normal, their total distribution (function distribution) will also differ from normal. In general, the type of total distribution is unknown. Probability theory allows us to approximate and in this case describe the distribution density function in the form of a Gram-Charlier series [19]. This will require statistical characteristics of the studied quantity up to the fourth central moment.

The principal provisions of the stated parametric theory of reliability in relation to the foundations of shallow foundations were developed by N.N.Ermolaev and V.V.Mikheev [12].

In fact, a comprehensive, theoretically more rigorous approach to solving problems of reliability of building structures under static and dynamic loads, taking into account the time factor, is considered in the works of V.V.Bolotin [11]. The mathematical basis of the method of its calculation is the theory of random functions, which considers failure as an outlier from a series of favorable events. At the same time, the patterns of distribution Y_1 и Y_2 they are assumed to be arbitrary, randomly changing over time. To solve practical problems according to this theory, organized statistical information is required, both about internal properties and various external factors, taking into account the time factor.



Currently, such information is practically absent in relation to building structures and foundations. When solving specific practical problems, both building structures and foundations usually do not include the time factor.

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