Interactions of a Circular Cylindrical Layer with a Viscous Incompressible Fluid

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Abstract
Circular cylindrical layers and shells of great length interacting with flows of liquids and gases. One of the important tasks of the dynamics of the "layer – liquid" and "shell – liquid" systems is to determine the frequencies and forms of natural oscillations.

Keywords: mechanical engineering, oil and gas production , aviation and space technology, inertia, deformation, fluctuations of a cylindrical elastic layer, longitudinal displacement, longitudinal deformation.

Circular cylindrical layers and shells of great length (tunnels, pipelines, drill strings) interacting with flows (both external and internal) of liquids and gases are widely used in a number of branches of science, technology and industry, including mechanical engineering, mining mechanics, oil and gas production , aviation and space technology, etc. [1,2].

One of the important tasks of the dynamics of the "layer-liquid" and "shell-liquid" systems is to determine the frequencies and forms of natural oscillations. In many works, such problems were solved by considering a cylindrical layer or a thick-walled pipe as a beam [3] or based on the theory of Kirchhoff–Love shells [4]. In [1], similar problems were solved on the basis of refined equations that take into account rotational inertia and transverse shear deformations.

This work is devoted to solving applied problems of axisymmetric oscillations of a circular cylindrical elastic layer interacting with viscous incompressible and ideal compressible fluids based on the oscillation equations obtained in the second chapter.

Restricting ourselves to the zero approximation (n = 0) in the general equations of torsional vibrations of a cylindrical layer containing a viscous incompressible fluid, we obtain

$$\frac{r_2^2}{2}\lambda_2 U_{\theta,0} + r_1\left[\frac{1}{2}\left(\lambda_2 - \frac{4}{r_2^2}\right) + \frac{r_2^2}{8}\left(\ln\frac{r_2}{r_1} - \frac{1}{4}\right)\lambda_2^2\right]U_{\theta,1} = \frac{1}{\mu} f_{r\theta}(z,t);$$

$$\frac{r_2^2}{2}\lambda_2 U_{\theta,0} + \frac{r_1^2}{4}\frac{\mu'}{v'\mu}\frac{\partial^2 U_{\theta,0}}{\partial t^2} + \left(\frac{2}{r_1} + \frac{r_1}{2}\lambda_2 - \frac{r_1^3}{32}\lambda_2^2\right)U_{\theta,1} + \frac{r_1}{4}\frac{\mu'}{v\mu}\frac{\partial^2 U_{\theta,1}}{\partial t^2} = 0, \quad (1)$$

where

$$\lambda_2^n = \left[\frac{1}{b^2}\left(\frac{\partial^2}{\partial t^2}\right) - \left(\frac{\partial^2}{\partial z^2}\right)\right]^n, \quad n = 0,1,2,...; \quad b = \sqrt{\frac{\mu}{\rho}}.$$

When solving problems of free vibrations, the layer surfaces are free from external loads, so the function $f_{r\theta}(z,t)$ on the right side of the first equation of system (1) will be considered equal to zero. The effect of the viscous incompressible fluid contained in the layer is taken into account by the second and fourth terms of the second equation of system (1). In system (1), we set $f_{r\theta}(z,t)=0$ and pass to dimensionless variables according to the formulas

$$U_{\theta,0}=U^{*}_{\theta,0};\quad U_{\theta,1}=r_1U^{*}_{\theta,1};\quad r=r_1r^*;\quad t=\frac{r_1}{b}t^*;\quad z=r_1z^*;\quad r_2=r_1r_2^*;\quad r_1^*=1 \qquad (1^*)$$

and for the convenience of notation, we omit the "asterisks" above the notation in the future. Get

$$\frac{r_2^2}{2}\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)U_{\theta,0}+\frac{1}{2}\left[\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}-\frac{4}{r_2^2}\right)+\frac{r_2^2}{4}\right]+\left[\frac{r_2^2}{8}\left(\ln\frac{r_2}{r_1}-\frac{1}{4}\right)\times\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)^2\right]U_{\theta,1}=0;$$

$$\frac{1}{4}\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)U_{\theta,0}+\frac{1}{4}\frac{\rho_0}{\rho}\frac{\partial^2U_{\theta,0}}{\partial t^2}+\left[2+\frac{1}{2}\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)-\frac{1}{32}\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)^2\right]U_{\theta,1}+$$

$$+\frac{1}{4}\frac{\rho_0}{\rho}\frac{\partial^2U_{\theta,0}}{\partial t^2}=0. \qquad (2)$$

Here ρ_0 and ρ are the densities of the unperturbed fluid and material of the cylindrical layer; $U_{\theta,0}$, $U_{\theta,1}$ the main parts of the torsional movement are the points of the layer. In the case of zero approximation, there is the following relationship between them

$$U_\theta=U_{\theta,1}+r_1U_{\theta,0},$$

which, after passing to dimensionless variables, takes the form

$$U_\theta=U_{\theta,1}+U_{\theta,0}. \qquad (3)$$

The solution of system (1) will be sought in the form

$$U_\theta=\overline{U}_{\theta,0}e^{i(\alpha z+\omega t)}$$

$$U_{\theta,1}=\overline{U}_{\theta,1}e^{i(\alpha z+\omega t)}. \qquad (4)$$

In this case, the dimensionless frequency ω and the wave number α are introduced by the formulas

$\omega^*=\dfrac{b}{r_1}\omega;$ $\alpha^*=r_1\alpha$; b – is the velocity of propagation of transverse waves in the layer.

We introduce the following notation

$$u=\frac{1}{4}\frac{\rho_0'}{\rho};\quad b_1=\frac{1}{16}\alpha^2-\frac{1}{2}-u;\quad b_2=2+\frac{1}{2}\alpha^2-\frac{1}{32}\alpha^4;\quad b_3=-\frac{1}{4}-u;$$

$$b_4=\frac{r_2^2}{2}\left(\ln r_2-\frac{1}{4}\right);\quad b_5=-1-\frac{r_2^2}{2}\left(\ln r_2-\frac{1}{4}\right)\alpha^2;\quad b_6=\alpha^2-\frac{4}{r_2^2}+\frac{r_2^2}{4}\left(\ln r_2-\frac{1}{4}\right)\alpha^4. \qquad (5)$$

In notation (5), the quantity U is a coefficient of the terms of the equation that take into account the effect of a viscous fluid on torsional vibrations of a cylindrical elastic layer. Substituting (5) into equation (1) we will have

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$$\left[\frac{r_2^2}{64}-b_3b_4\right]\omega^6+\left[-\frac{1}{64}\alpha^2r_2^2-b_1\frac{r_2^2}{2}-\frac{1}{4}\alpha^2b_4-b_3b_5\right]\omega^4+$$

$$+\left[b_1\frac{r_2^2}{2}\alpha^2-b_3\frac{r_2^2}{2}-\frac{1}{4}\alpha^2b_5-b_3b_6\right]\omega^2+\left[\frac{b_2}{2}\alpha^2r_2^2+\frac{1}{4}\alpha^2b_6\right]=0. \qquad (6)$$

Let us introduce further notation by the formulas

$$a_6=\frac{r_2^2}{64}-b_3b_4; \quad a_4=-\frac{1}{64}\alpha^2r_2^2-b_1\frac{r_2^2}{2}-\frac{1}{4}\alpha^2b_4-b_3b_5;$$

$$a_2=b_1\frac{r_2^2}{2}\alpha^2-b_3\frac{r_2^2}{2}-\frac{1}{4}\alpha^2b_5-b_3b_6; \quad a_0=\frac{b_2}{2}\alpha^2r_2^2+\frac{1}{4}\alpha^2b,$$

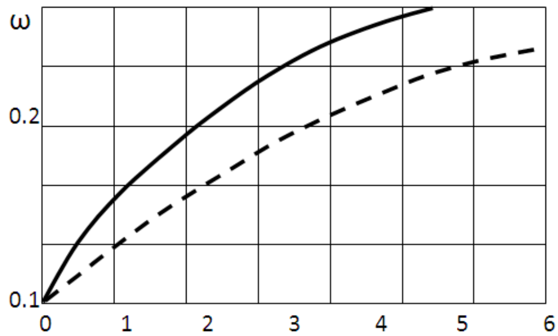
taking into account which the frequency equation (6) can be rewritten in the form convenient for numerical implementation

$$a_6\omega^6+a_4\omega^4+a_2\omega^2+a_0=0. \qquad (7)$$

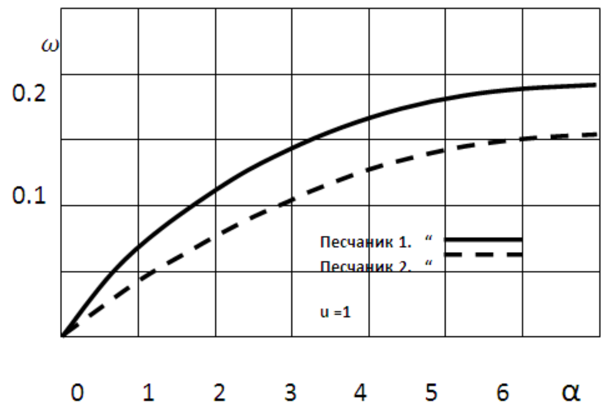
To solve this equation, Horner's scheme was applied, a Pascal program was compiled and implemented on a PC (application).

For the numerical study of the problem, rocks were used as the layer material.

On fig. 1 and 2 show the same dependences for sandstones 1, 2. It can be seen from the graphs that the oscillation frequency P.1 is always greater than the oscillation frequency P.2, which in turn is much higher than the oscillation frequency P.3, as in the case in the absence (u = 0, Fig. 2) and in the presence (u = 1, Fig. 1) of liquid inside the layer cavity. Can do



Rice. 1. Dependences of the first frequencies on the wave number for absence of liquid (u = 0)



Rice. 2. Dependences of the first frequencies on the wave number c liquid (u=1)

On fig. 1 and 2 show the same dependences for silts 1, 2. It can be seen from the graphs that the oscillation frequency of item 1 is always greater than the oscillation frequency of item 2, which, in turn, is much higher than the oscillation frequency of item 3, both in the case of absence (u = 0, Fig. 2) and in the case of the presence (u = 1, Fig. 1) of liquid inside the cavity of the layer. It can be concluded that with an increase in the value of the Poisson's ratio and the density of the rock, the frequency of its oscillations decreases.

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